5.3 The Graph of a Rational Function

Learning Objectives

1. Analyze the Graph of a Rational Function
2. Solve Applied Problems Involving Rational Functions

Example

Analyzing the Graph of a Rational Function

Analyze the graph of the rational function:

$$R(x) = \frac{x^2 - 4}{x + 1} = \frac{(x+2)(x-2)}{x+1}$$

Find the domain, intercepts, and asymptotes of the function.

Domain: $\{x \mid x \neq -1\}$

V.A: $x = -1$

H.A: none

O.A: $\boxed{y = x - 1}$

$$\begin{array}{r} x - 1 \\ x+1 \overline{) x^2 + 0x - 4} \\ -(x^2 + x) \\ \hline -1x - 4 \\ -(-1x - 1) \\ \hline \end{array}$$

remainder -3

Intercepts

$$R(x) = \frac{x^2-4}{x+1}$$

To find y-int. let $x=0$...

$$R(0) = \frac{0^2-4}{0+1} = -4$$

$(0,-4)$

To find x-int(s), let $y=0$

$$\cancel{(x+1)}\,\frac{x^2-4}{\cancel{x+1}} = 0\,(x+1)$$

$$x^2-4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

$(\pm 2, 0)$

Example

Analyzing the Graph of a Rational Function

Analyze the graph of the rational function: $R(x) = \dfrac{3x^2 - 3x}{x^2 + x - 12}$

$= \dfrac{3x(x-1)}{(x+4)(x-3)}$

Domain: $\{x \mid x \neq -4, 3\}$

V.A: $x = -4, 3$

H.A: $y = 3$

O.A: none

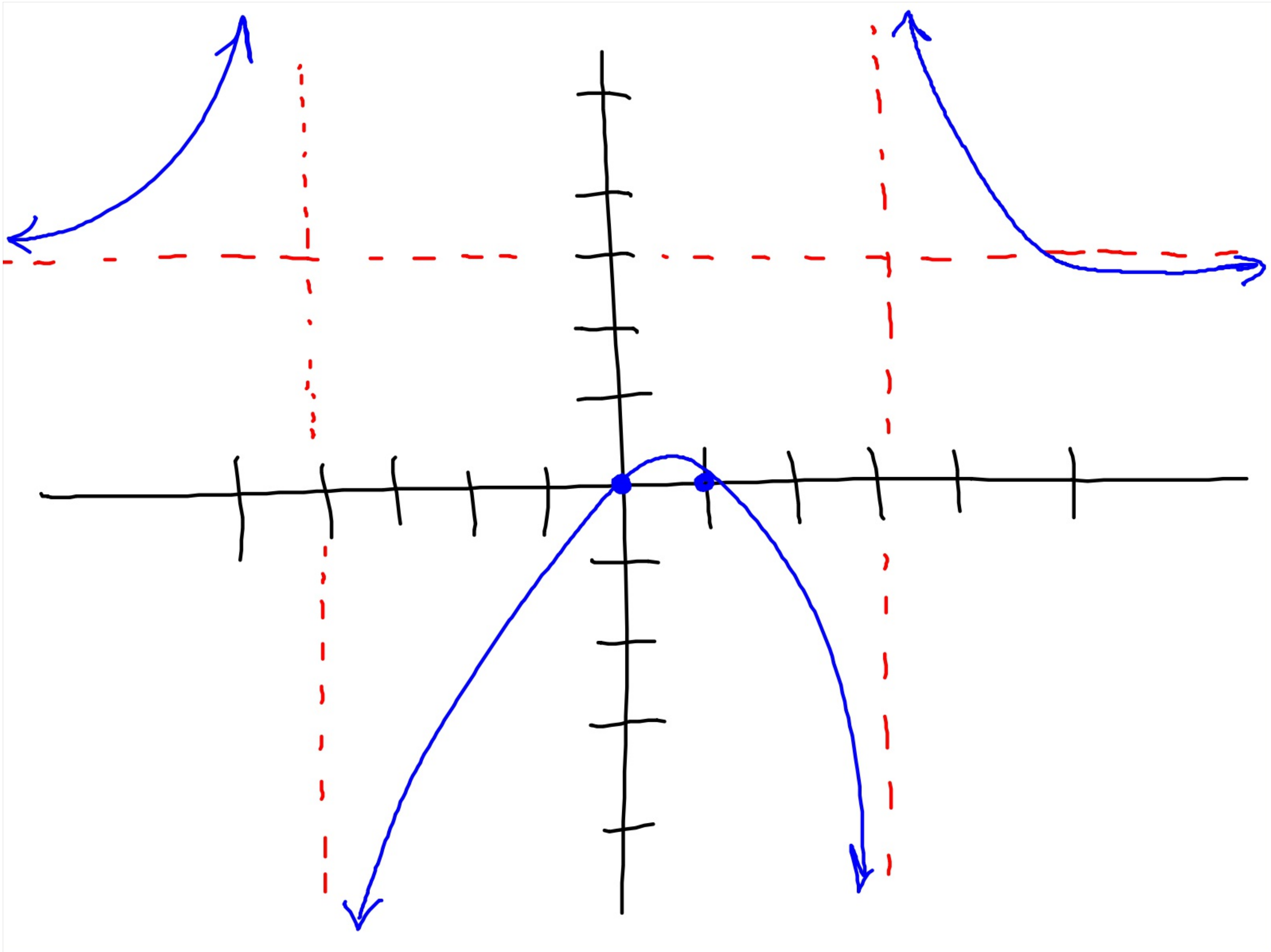
y-int: $(0, 0)$

x-int(s): (zeros of numerator!) $\rightarrow (0,0)\ (1,0)$

$\cancel{(x^2+x-12)}\ \dfrac{3x^2-3x}{\cancel{x^2+x-12}} = 0\ (x^2+x-12)$

$3x^2 - 3x = 0$

$3x(x-1) = 0 \rightarrow x = 0, 1$



Example

Analyzing the Graph of a Rational Function with a Hole

Analyze the graph of the rational function: $R(x) = \dfrac{2x^2 - 5x + 2}{x^2 - 4}$

$$= \frac{(2x-1)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}$$

$$= \frac{2x-1}{x+2}$$

"hole" at $x=2$

Domain: $\{x \mid x \neq \pm 2\}$

V.A.: $x = -2$

H.A: $y = 2$

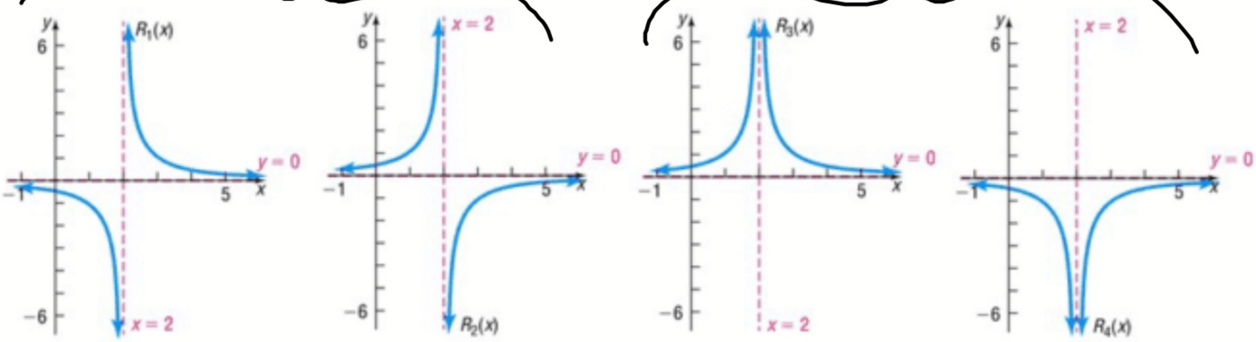
O.A: none

y-int: $\left(0, -\frac{1}{2}\right)$

X-int(s): $\left(\frac{1}{2}, 0\right)$

Multiplicity and Vertical Asymptotes

odd mult.   even mult.



(a) Odd multiplicity
$\lim_{x\to 2^-} R_1(x) = -\infty$
$\lim_{x\to 2^+} R_1(x) = \infty$

(b) Odd multiplicity
$\lim_{x\to 2^-} R_2(x) = \infty$
$\lim_{x\to 2^+} R_2(x) = -\infty$

(c) Even multiplicity
$\lim_{x\to 2^-} R_3(x) = \infty$
$\lim_{x\to 2^+} R_3(x) = \infty$

(d) Even multiplicity
$\lim_{x\to 2^-} R_4(x) = -\infty$
$\lim_{x\to 2^+} R_4(x) = -\infty$

$$f(x) = \frac{1}{x-2}$$

$$f(x) = \frac{1}{(x-2)^2}$$

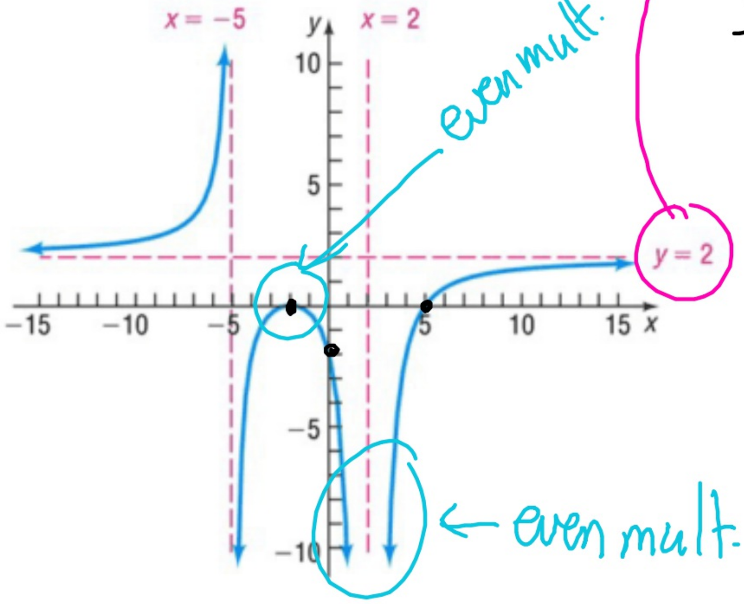
Example – Find the rational model for the graph.

x-ints

$$f(x) = \frac{2(x+2)^2(x-5)}{(x+5)(x-2)^2}$$

vert. asym.

even mult.



even mult.

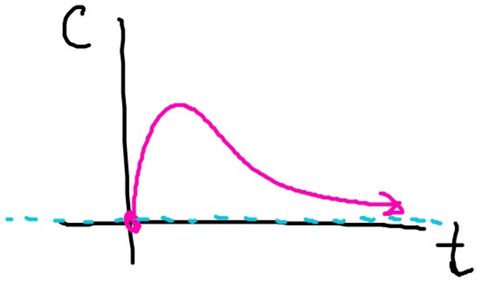
Example

The concentration C of a certain drug in a patient's bloodstream t hours after injection is given by: $C(t) = \frac{t}{2t^2+1}$.

What happens to the concentration of the drug as t increases?

as t increases, C decreases (approaches zero)

Notice the H.A. is $y=0$ →



Example

Your company has been contracted to make a cardboard box with a volume of 5000 square inches. The base is a square with sides *x*, and the height of the box is *y*. You want to use the least amount of cardboard to save money. Find the minimum surface area of the box that still has the required volume.

$$V = x \cdot x \cdot y$$

$$V = x^2 y$$

$$5000 = x^2 y$$

$$\frac{5000}{x^2} = y$$

$$SA = 2x^2 + 4xy$$

$$SA = 2x^2 + 4x\left(\frac{5000}{x^2}\right)$$

$$\boxed{SA = 2x^2 + \frac{20000}{x}}$$

now minimize the surface area function by graphing it using technology... you will find the local min. @ $x = 17$